Math 115
Winter 2017
Lecture 8

Find equation of a line that contains $(2,-4)$ and $(4,1)$.

① $m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{-4-1}{2-4} = \frac{-5}{-2} = \frac{5}{2}$

② $y - y_1 = m(x - x_1)$

$y - 1 = \frac{5}{2}(x-4)$

$y - 1 = \frac{5}{2}x - \frac{5}{2}\cdot 4^2$

$y = \frac{5}{2}x - 10 + 1$

$y = \frac{5}{2}x - 9$

$m = \frac{5}{2}$

Y-Int $(0,-9)$

Find eqn of a line that contains $(-3,2)$ and is parallel to $4x-3y=9$.

Same slope

$-3y=-4x+9$

$y=\frac{4}{3}x-3$

$m=\frac{4}{3}$

$y-y_1=m(x-x_1)$

$y-2=\frac{4}{3}(x+3)$

$y-2=\frac{4}{3}x+4$

$y=\frac{4}{3}x+6$

Find eqn of a line that contains $(2,-5)$ and is perpendicular to the line $3x+4y=8$.

$m_1 \cdot m_2=-1$

check $\frac{4}{3}\cdot\frac{-3}{4}=-1$ ✓

$4y=-3x+8$

$y=\frac{-3}{4}x+2$

our line has a slope of $\frac{4}{3}$.

$y-y_1=m(x-x_1)$

$y--5=\frac{4}{3}(x-2)$

$y+5=\frac{4}{3}(x-2)$

$3y+15=4(x-2)$

$3y+15=4x-8$

$3y=4x-23$

$y=\frac{4}{3}x-\frac{23}{3}$

Graph

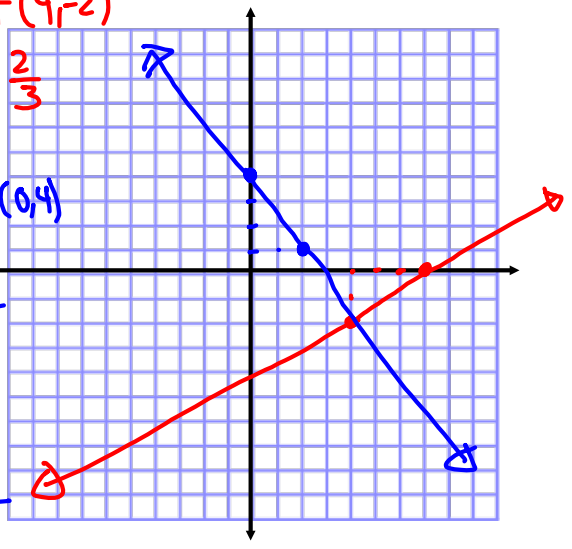
$\begin{cases} y+2=\frac{2}{3}(x-4) \\ y=\frac{-3}{2}x+4 \end{cases}$

Point $(4,-2)$

$m=\frac{2}{3}$

Y-Int $(0,4)$

$m=\frac{-3}{2}$



Since $\frac{2}{3}\cdot\frac{-3}{2}=-1$

these lines are $\perp$

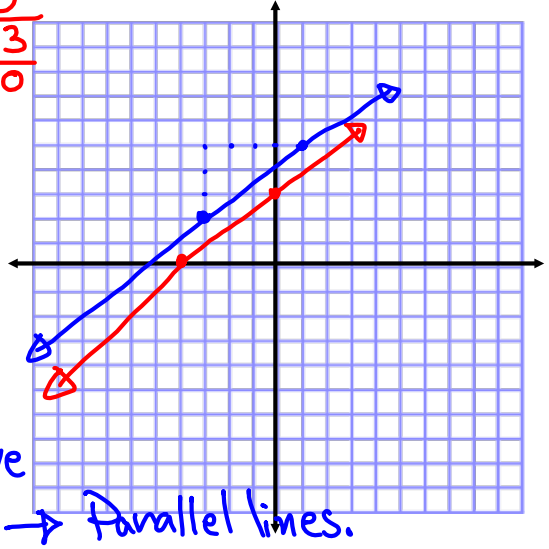
Graph

$\begin{cases} 3x-4y=-12 \\ y-2=\frac{3}{4}(x+3) \end{cases}$

x	y
0	3
-4	0

Point $(-3,2)$

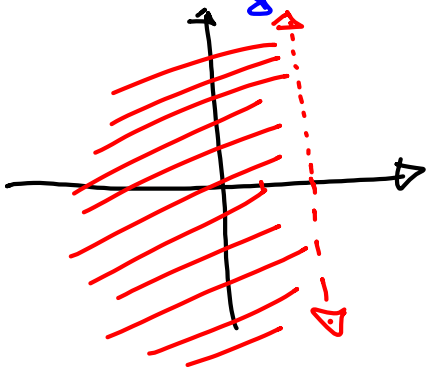
$m=\frac{3}{4}$



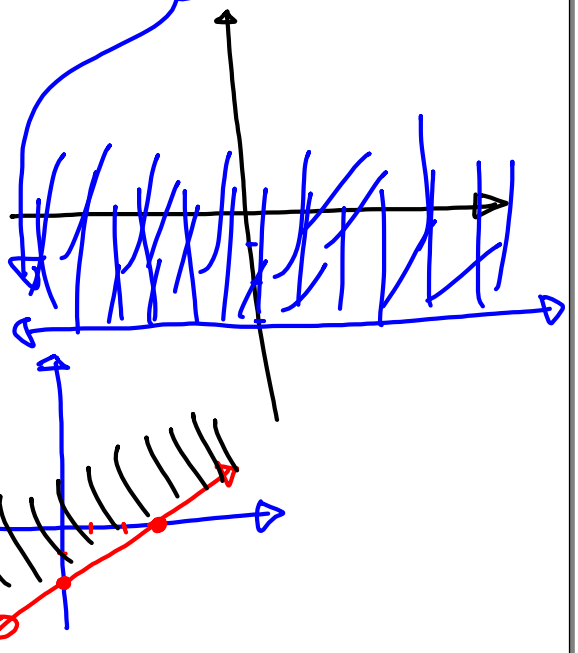
Since these lines have Same slope → Parallel lines.

Graph & Shade

① $x < 2$



② $y \geq -3$



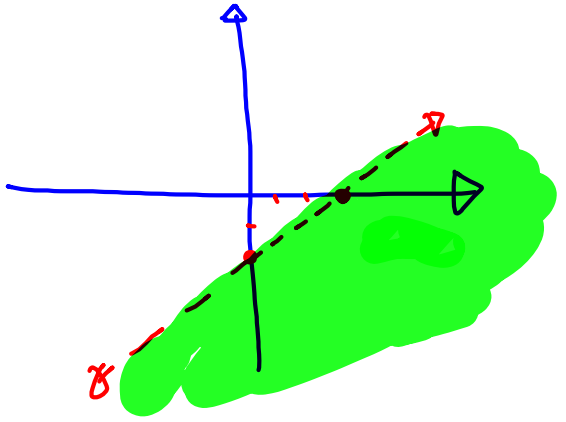
③ $y \geq \frac{2}{3}x - 2$

④ $2x - 3y > 6$

Hint: write in Slope-Int form.

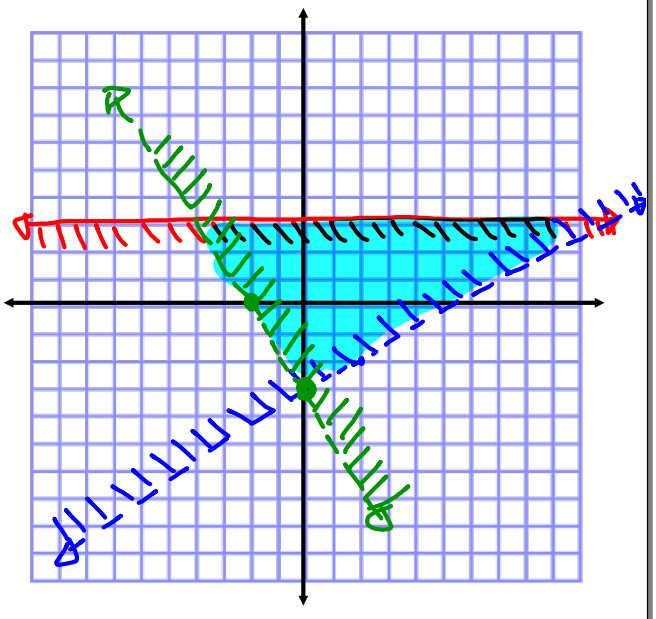
$-3y > -2x + 6$

$y < \frac{2}{3}x - 2$



Shade the Solution.

$$\begin{cases} y \le 3 \\ y > \frac{1}{2}x - 3 \\ 3x + 2y > -6 \end{cases}$$



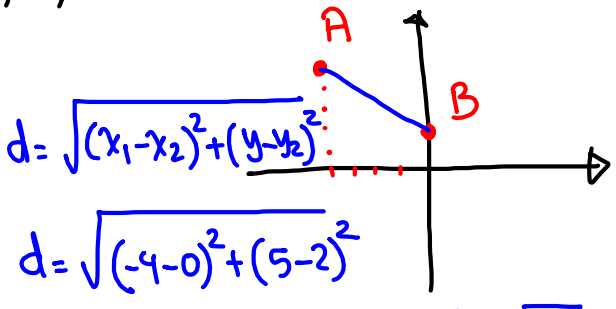
A(-4,5) , B(0,2)

① Draw $\overline{AB}$

② Find d(A,B)

③ Find midpoint

④ Find slope



$d = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

$d = \sqrt{(-4-0)^2+(5-2)^2}$

$= \sqrt{(-4)^2+3^2} = \sqrt{25} = \boxed{5}$

$M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{-4+0}{2}, \frac{5+2}{2}\right)$

$M(-2, 3.5)$

$m = \frac{y_1-y_2}{x_1-x_2} = \frac{5-2}{-4-0} = \frac{3}{-4} = \boxed{\frac{-3}{4}}$

System of linear equations in two Variables

$\begin{cases} x+y=4 \\ x-y=2 \end{cases}$, $\begin{cases} x+y=17 \\ y=1-x \end{cases}$

$\begin{cases} 2x-3y=6 \\ 3x+4y=-10 \end{cases}$, $\begin{cases} y=\frac{2}{3}x-8 \\ y=-2x+4 \end{cases}$

$\begin{cases} 2x+y=12 \\ .05x-.25y=20.7 \end{cases}$

If there is a Solution, Soln has to be in the form of ordered-Pairs.

Soln has to make both equations a true statement.

Is $(2,-3)$ a Solution of $\begin{cases} x+y=-1 \\ 3x+2y=0 \end{cases}$?

$x+y=-1$

$2+(-3)=-1$

$-1=-1$ ✓

$3x+2y=0$

$3(2)+2(-3)=0$

$6+(-6)=0$

$0=0$ ✓

Yes, $(2,-3)$ is a Soln.

Is $(-3,4)$ a Solution of

$$\begin{cases} 2x+3y=6 \\ 3x-y=13 \end{cases} ?$$

$(-3,4) \rightarrow x=-3,\ y=4$

$3x - y = 13$

$3(-3) - 4 = 13$

$-9 - 4 = 13$

$-13 = 13$ ← false

$2x + 3y = 6$

$2(-3) + 3(4) = 6$

$-6 + 12 = 6$

$6 = 6$ ✓

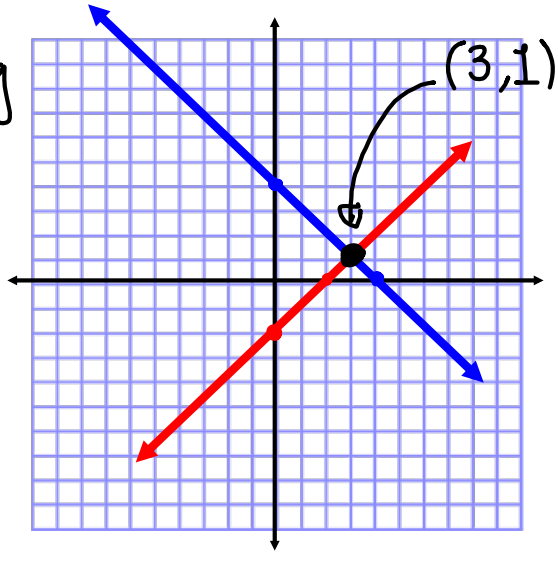
So $(-3,4)$ is not a Soln.

We can Solve the system by different methods.

① Graphing ② Substitution ③ Addition

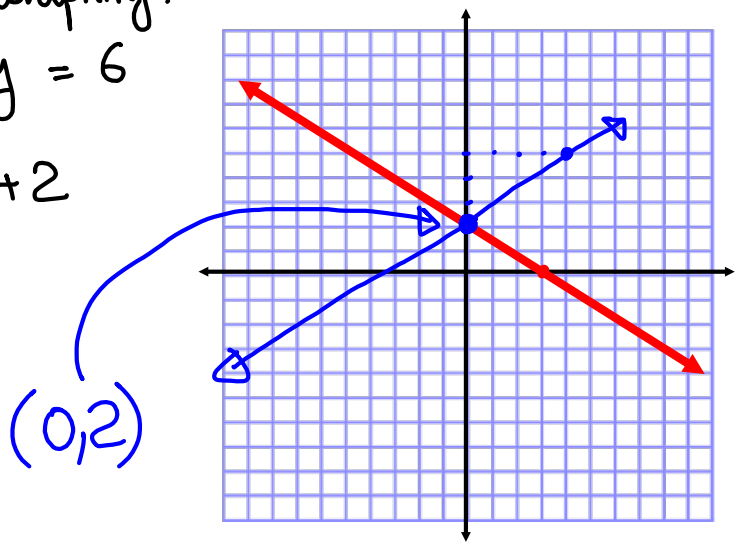
Solve by graphing

$$\begin{cases} x+y=4 \\ x-y=2 \end{cases}$$



Solve by Graphing:

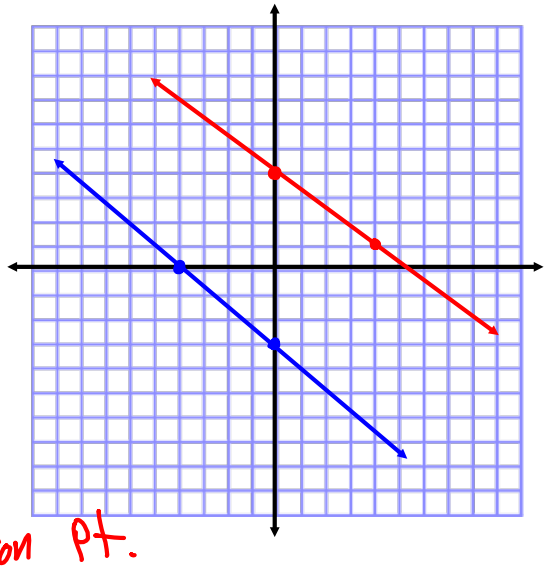
$$\begin{cases} 2x + 3y = 6 \\ y = \frac{3}{4}x + 2 \end{cases}$$



(0,2)

Solve by Graphing:

$$\begin{cases} 3x + 4y = -12 \\ y = \frac{-3}{4}x + 4 \end{cases}$$



Same Slope

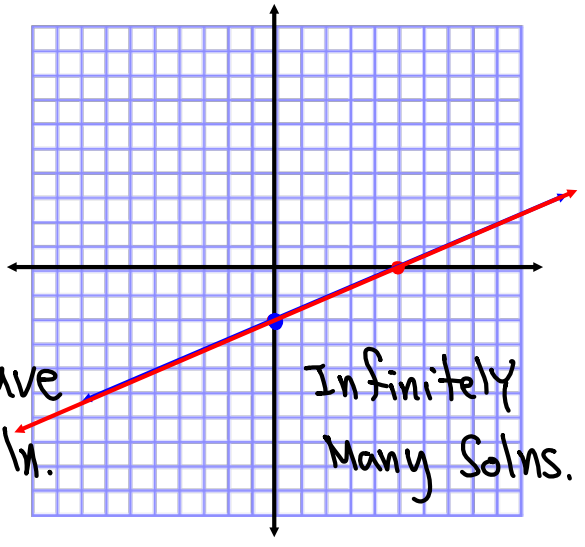
⇒ Parallel lines

⇒ No intersection pt.

⇒ $\varnothing$ No Solution

Solve by graphing:

$$\begin{cases} 2x - 5y = 10 \\ y = \frac{2}{5}x - 2 \end{cases}$$



So, we may have

- exactly one Soln.
- NO Soln
- Infinitely Many Solns.

Solve by Substitution:

$$\begin{cases} x + y = 7 \\ y = 2x - 2 \end{cases}$$

$x + (2x - 2) = 7$

$3x - 2 = 7$

$3x = 9 \rightarrow x = 3$

$y = 2(3) - 2$

$y = 6 - 2$

$y = 4$

$(3, 4)$ ← final Ans

$\{(3, 4)\}$

Solve by Subs.

$$\begin{cases} y = 5 - 2x \\ 3x - y = -10 \end{cases}$$

$3x - (5 - 2x) = -10$

$3x - 5 + 2x = -10$

$5x = -10 + 5$

$5x = -5 \rightarrow \boxed{x = -1}$

$y = 5 - 2x$

$y = 5 - 2(-1)$

$\boxed{y = 7}$

$\{(-1, 7)\}$

Solve by Subs.

$$\begin{cases} x - 2y = 7 \\ y = \frac{1}{2}x - 3 \end{cases}$$

$x - 2\left(\frac{1}{2}x - 3\right) = 7$

$x - 2 \cdot \frac{1}{2}x + 2 \cdot 3 = 7$

$x - x + 6 = 7$

$6 = 7$

False

Solve by Subs.

$$\begin{cases} 4x - 3y = 9 \\ y = \frac{4}{3}x - 3 \end{cases}$$

$\Rightarrow 4x - 3\left(\frac{4}{3}x - 3\right) = 9$

$4x - 3 \cdot \frac{4}{3}x + 3 \cdot 3 = 9$

$4x - 4x + 9 = 9$

$9 = 9$

True

Infinitely Many Solns.

Solve by Addition (Elimination):

$$\begin{cases} x + y = 4 \\ x - y = 2 \end{cases}$$

$2x = 6$

$\boxed{x = 3}$

$3 + y = 4$

$\boxed{y = 1}$

$\{(3, 1)\}$

$$\begin{cases} 3x + 2y = 2 \\ 2x - y = 6 \end{cases}$$ Solve by Addition.

$2 \times$ second equation:

$$\begin{cases} 3x + 2y = 2 \\ 4x - 2y = 12 \end{cases}$$

$$7x = 14$$

$$x = 2$$

$3x + 2y = 2$

$3(2) + 2y = 2$

$6 + 2y = 2$

$2y = -4$

$y = -2$

$\{(2, -2)\}$

$$\begin{cases} 3x + 2y = 7 \\ 5x - 3y = -1 \end{cases}$$

$3 \times$ first equation, $2 \times$ second equation:

$$\begin{cases} 9x + 6y = 21 \\ 10x - 6y = -2 \end{cases}$$

$$19x = 19$$

$$x = 1$$

$3x + 2y = 7$

$3(1) + 2y = 7$

$3 + 2y = 7$

$2y = 4$

$y = 2$

$\{(1, 2)\}$

Solve by addition

$$\begin{cases} 3x - 2y = 2 \\ 2x - 3y = -7 \end{cases}$$

$$\begin{cases} 9x - 6y = 6 \\ -4x + 6y = 14 \end{cases}$$

$$5x = 20$$

$x = 4$

$3(4) - 2y = 2$

$12 - 2y = 2$

$-2y = -10$

$y = 5$

$\{(4, 5)\}$

Sum of two numbers is 12.

Twice one of them reduced by

3 times the other one is equal to -1.

use system of linear equations to find both numbers.

Let x & y be the two numbers,

$$3\begin{cases} x + y = 12 \\ 2x - 3y = -1 \end{cases}$$

$$\begin{cases} 3x + 3y = 36 \\ 2x - 3y = -1 \end{cases}$$

$$5x = 35$$

$x = 7$

$7 + y = 12$

$y = 5$

The numbers are 5 & 7

The Perimeter of a rectangular Pool is 54 ft.

Its length is 1 ft shorter than 3 times its width. Use System of linear equations to find the dimensions of the pool.

$$\begin{cases} 2L + 2W = 54 \\ L = 3W - 1 \end{cases}$$

$$2(3W-1) + 2W = 54$$

$$6W - 2 + 2W = 54$$

$$8W = 56$$

$$W = 7$$

7 ft by 20 ft

PTA purchased 15 Tkts.

PTA Paid $78

Kid's tkt → $4

Adult's tkt → $10

How many of each? use System of linear equations.

A → # of adults

K → # of kids

$$\begin{cases} A + K = 15 \\ 10A + 4K = 78 \end{cases} \quad -4$$

$$\begin{cases} -4A - 4K = -60 \\ 10A + 4K = 78 \end{cases}$$

$$6A = 18$$

$$A = 3$$

3 Adults & 12 kids

Lisa has $1.75 in nickels & Dimes only. The number of nickels is 3 more than twice the number of dimes. Use system of linear equations to find the # of dimes.

N → Nickels

D → Dimes

$\div 5 \begin{cases} 5N + 10D = 175 \\ N = 2D + 3 \end{cases}$ $\begin{cases} N + 2D = 35 \\ N = 2D + 3 \end{cases}$

8 Dimes

$2D + 3 + 2D = 35$

$4D = 32$

$D = 8$

Jose needs 50 lb of candy @ $1.55/lb. He has two type of candies, one @ $1.25/lb and another one @ $1.75/lb. How much of each should he mix to obtain what he needs? Use system of linear equations.

$1.25/lb	+	$1.75/lb	=	$1.55/lb
x		y		50

$\begin{cases} x + y = 50 \\ 1.25x + 1.75y = 1.55(50) \end{cases}$ ×100

$\div 25 \begin{cases} x + y = 50 \\ 125x + 175y = 155(50) \end{cases}$

$$\begin{cases} x + y = 50 \\ 5x + 7y = 310 \end{cases} \rightarrow y = 50 - x$$

$$5x + 7(50 - x) = 310$$

$$5x + 350 - 7x = 310$$

$$-2x = 310 - 350$$

$$-2x = -40$$

$$\boxed{x = 20} \rightarrow \boxed{y = 30}$$

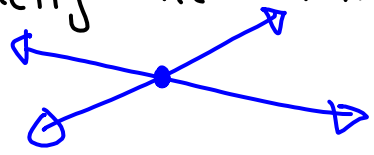
20 lb. @ $1.25
&
30 lb. @ $1.75

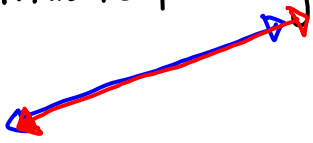
John sold 37 drinks in his shift.
He collected $67 in total.
Small drinks → $1.60, Large drinks → $2.25
How many of each? use S & L.

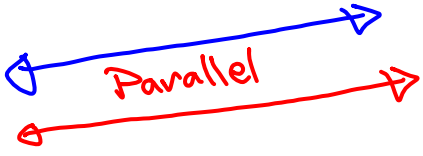
$$\begin{cases} S + L = 37 \\ 1.60S + 2.25L = 67 \end{cases} \times 100 \Rightarrow \begin{cases} S + L = 37 \\ 160S + 225L = 6700 \end{cases}$$

12 Large & 25 Small

When system of linear equations has

1) exactly one Solution ⇒ System is Consistent

⇒ Equations are independent

2) infinitely many Solutions ⇒ System is Consistent

⇒ Equations are dependent

3) has no Solution ⇒ System is inconsistent


⇒ Equations are independent

Solve by graphing

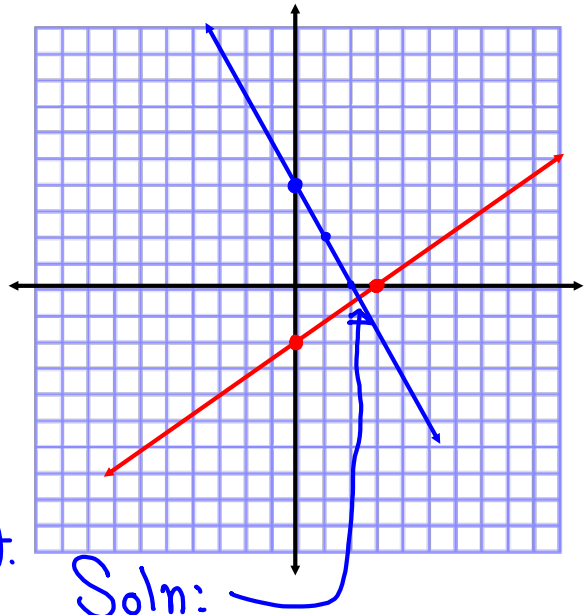
$$\begin{cases} 2x - 3y = 6 \\ y = -2x + 4 \end{cases}$$

Y-Int (0,4)

$m = -2 = \frac{-2}{1}$

System is Consistent.
Eqns are independent.

Graphing method is not a prefered choice.



Soln: Not a nice Soln.

Solve

$\begin{cases} x = 3y - 5 \\ x = y + 9 \end{cases}$

whenever a variable is isolated, use Subs.

$3y - 5 = y + 9$

$3y - y = 9 + 5$

$2y = 14$

$y = 7$

$x = 16$

$\Rightarrow (16, 7)$

System: Consistent

Eqns: independent.

Due Wednesday

① wp 6

② SG 11

Expect a Quiz @ 6:00 AM.